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Supercritical resonance in a strong magnetic field

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The position and width of supercritical resonance of one-electron ion in presence of a strong magnetic field are calculated. The influence of the magnetic field on the supercritical resonance parameters is investigated.

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Introduction

Quantum electrodynamics predicts that qualitatively new effects caused by vacuum instability shall be observed in a ultrastrong Coulomb field [1–6]. In particular, as shown in [2,3], when nuclear charge exceeds the critical value $Z_{\mathrm{cr}} \gtrsim 173$, the ground state of a hydrogen-like ion is immersed in the negative continuum and becomes a resonance state. As a result, with a probability according to the resonance width, vacuum decays to form a positron. Despite the fact that there are no nuclei with a near-critical charge, recent theoretical articles have demonstrated that spontaneous vacuum decay may be observed in the experiments with slow collision of heavy ions whose cumulative charge exceeds the critical charge [7,8]. However, such observation is a difficult task in terms of experiment. Therefore, it is reasonable to consider increase of the supercritical resonance width and hence a probability of vacuum decay using an additional external field.

Behavior of the ground state of a hydrogen-like ion put into an ultrastrong uniform magnetic field is addressed herein. As shown in [9,10], the presence of such field effectively reduces the critical nuclear charge Z_{cr} with which the ground state is immersed in the negative energy continuum. This is due to the fact that the electron wave function in the ground state „contracts“ and becomes quasi-one-dimensional which enhances the Coulomb potential singularity. For the same reason, it may be expected that the presence of magnetic field will increase the probability of spontaneous vacuum decay. The main purpose of the research is to study this effect.

Earlier in [9–12], dependence of Z_{cr} on the uniform magnetic field strength was studied. In addition, electron ground state energy calculations were carried out, but only until the immersion into the negative energy continuum and transformation into resonance. It should be noted that [11,12] also were focused on the influence of the Coulomb nuclear field screening which occurs in an ultra-strong magnetic field on the critical charge value.

A complex-coordinate rotation method is used herein in order to obtain the resonance state parameters. This method involves rotation of a radial coordinate to a complex plane. As a result, resonance wave functions become square integrable and the corresponding energies have complex values, the real part of which determines the resonance position, and the imaginary part determines the resonance width. In [13–16], the complex-coordinate rotation technique was successfully used to calculate the ultrastrong resonance parameters in a Coulomb field. For detailed description of the complex-coordinate rotation method and its numerous variations, see [17,18]. Ground state energy of a one-electron ion after immersion into the negative energy continuum in the presence of magnetic field has been obtained herein using the complex-coordinate rotation technique. Supercritical resonance width has been also calculated depending on the magnetic field strength which enabled to estimate its influence on the spontaneous vacuum decay probability.

Atomic system of units is used herein: $\hbar = 1$, $m = 1$, $e = -1$. In such units, the speed of light in vacuum is equal to $1/\alpha \approx 137.036$, where α is the thin structure constant. In the calculation results, magnetic field strength is given in the units of critical strength $B_0 = \frac{m^2c^3}{\hbar|e|}$, which is related to SI units as $c^{-1}B_0 \approx 4.41 \cdot 10^9$ T.

Theory and calculation methods

In presence of a permanent magnetic field, the electron wave function in a hydrogen-like system satisfies the Dirac steady-state equation:

$$H_0\Psi(\mathbf{r}) = E\Psi(\mathbf{r}), \tag{1}$$

where Hamiltonian

$$H_0 = c\boldsymbol{\alpha} \cdot \left(\mathbf{p} + \frac{\mathbf{A}}{c}\right) + c^2\beta + V_{\mathrm{nucl}}$$

$$= c\boldsymbol{\alpha} \cdot \mathbf{p} + c^2\beta + V_{\mathrm{nucl}} + V_{\mathrm{ext}}. \tag{2}$$

Here, $c \approx 137$ is the speed of light in vacuum, $\boldsymbol{\alpha}$ and β are Dirac matrices. A spherically symmetrical potential $V_{\text{nucl}}(r)$ describes interaction with the nucleus. Unless otherwise specified explicitly, then a uniformly charged sphere model is used in the calculations:

$$V_{\text{nucl}}(r) = \begin{cases} -\frac{Z}{r}, & r \geqslant R_{\text{nucl}}, \\ -\frac{Z}{2r}\left(3 - \frac{r^2}{R_{\text{nucl}}}\right), & r < R_{\text{nucl}}. \end{cases} \tag{3}$$

Potential $V_{\text{ext}}(\mathbf{r})$ describes interaction between the electron with the permanent magnetic field directed along z axis. Chose a vector potential in the following form $\mathbf{A} = \frac{1}{2}[\mathbf{B} \times \mathbf{r}]$, then

$$V_{\text{ext}}(\mathbf{r}) = \frac{1}{2}\mathbf{B} \cdot [\mathbf{r} \times \boldsymbol{\alpha}]. \tag{4}$$

Consider the Dirac equation (1) in the spherical coordinate system (r, θ, φ). Due to the axial Hamiltonian symmetry (2), the total electron momentum projection $m_j = -1/2$ on z axis will be constant and the solution may be sought for in the following form:

$$\Psi(r, \theta, \varphi) = \frac{1}{\sqrt{2\pi} r} \begin{pmatrix} \mathrm{e}^{-i\varphi}\psi_1(r, \theta) \\ \psi_2(r, \theta) \\ i\mathrm{e}^{-i\varphi}\psi_3(r, \theta) \\ i\psi_4(r, \theta) \end{pmatrix}. \tag{5}$$

A four-component function $\psi(r, \theta)$ composed of scalar functions $\psi_i(r, \theta)$ satisfies the following equation:

$$\widetilde{H}\psi(r, \theta) = E\psi(r, \theta), \tag{6}$$

where

$$\widetilde{H} = \begin{pmatrix} (V_{\text{nucl}} + c^2) \cdot \mathbf{1}_2 & cD \\ -cD & (V_{\text{nucl}} - c^2) \cdot \mathbf{1}_2 \end{pmatrix} \tag{7}$$

and

$$D = \frac{1}{2c} Br \sin\theta i\sigma_y + (\sigma_x \sin\theta + \sigma_z \cos\theta)\left(\frac{\partial}{\partial r} - \frac{1}{r}\right) + \frac{1}{r}(\sigma_x \cos\theta - \sigma_z \sin\theta)\frac{\partial}{\partial \theta} + \frac{1}{2r \sin\theta}(\sigma_x - i\sigma_y). \tag{8}$$

Here, σ_x, σ_y and σ_z are Pauli matrices, $\mathbf{1}_2$ is the single matrix $2 \cdot 2$.

To find resonance solutions of the equation (6), the complex-coordinate rotation method where a radial coordinate r is „rotated" at the angle $\Theta(r)$ to a complex plane:

$$\tilde{r} = re^{i\Theta(r)}. \tag{9}$$

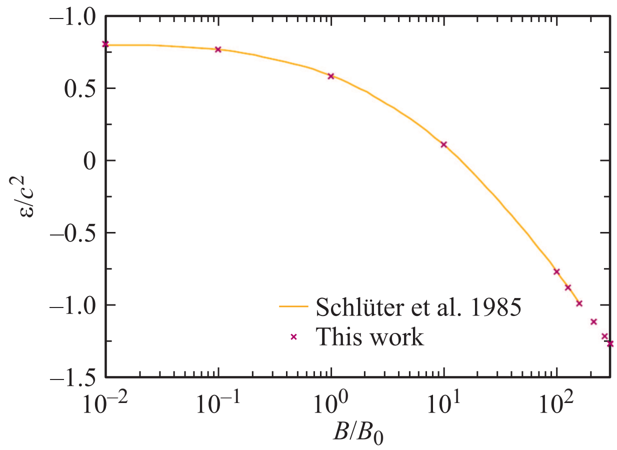
Dependence of Θ on r enables to rotate the whole axis and its part. The use of various parameters that determine the form of function $\Theta(r)$ enables to improve the convergence of the obtained results.

Hamiltonian $\widetilde{H}$ after rotation is not Hermitian any longer and the level energy E takes complex values. It is convenient to write it as follows

$$E = \varepsilon + i\Gamma/2, \tag{10}$$

Resonance state position and width $1s_{-1/2}$ of the hydrogen-like ion with $Z = 178$ placed in the uniform magnetic induction field B ($c^{-1}B_0 \approx 4.41 \cdot 10^9$ T)

B/B_0	ε/c^2	Γ/c^2
0	−1.563	$3.4 \cdot 10^{-3}$
0.053	−1.566	$3.7 \cdot 10^{-3}$
0.1	−1.569	$4.0 \cdot 10^{-3}$
0.266	−1.578	$4.4 \cdot 10^{-3}$
0.4	−1.587	$4.7 \cdot 10^{-3}$



Real part of the energy of state $1s_{-1/2}$ of the hydrogen-like lead ion ($Z = 82$) depending on induction B of the uniform magnetic field ($c^{-1}B_0 \approx 4.41 \cdot 10^9$ T).

where ε is the resonance position, and Γ is the resonance width. Equation (6) can be discretized using the generalized pseudo-spectral method [19,20] within which it is reduced to a complex-valued matrix eigenproblem. For our calculations, we use FEAST library [21] that allows efficient search of sparse matrix eigenvalues lying within the pre-defined contour in a complex plane.

Results and discussion

In order to check the performance of the method described above, the ground state energy of the hydrogen-like lead ion ($Z = 82$) place in the uniform magnetic field was calculated for various magnetic field values B. The results are shown in the figure in comparison with the corresponding values from [10]. To compare the results with the previous ones, a point nucleus model was used here. The curve shows that the obtained results match well with the corresponding values from [10]. And the field of about 10^{11} T brings this ion into a supercritical mode (i. e. the ground state is immersed in the negative energy continuum). The complex-coordinate rotation method used herein made it possible to track the ground state position after the immersion which is also displayed on the curve.

A hypothetical ion with nuclear charge $Z = 178$ was used as an example of a system which is supecritical even without any magnetic field. With such nuclear charge, the ground state is already built in the negative energy continuum without any external field and has a form of resonance with some nonzero width. In order to study the magnetic field effect, superctitical resonance position and width were calculated for several field strengths B. The results are shown in the Table.

It can be seen that the resonance position is shifted down as the field strength grows. And the level width also grows which is indicative of increased probability of spontaneous vacuum decay. Significant change in this probability takes place at magnetic field strength B which is significantly lower than the critical value B_0. Although the described values of B are far from the experimentally accessible ones, the obtained results show the probability of signal amplification by the spontaneous vacuum decay in supercritical heavy ion collisions in the presence of indicative of magnetic field.

Conclusion

The ground state position and width of the supercritical one-electron ion in the presence of uniform magnetic field have been calculated. The calculation has been made by the complex-coordinate rotation method using the finite basis set. The probability of spontaneous vacuum decay in the presence of magnetic field has been demonstrated.

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Conflict of interest

The authors declare that they have no conflict of interest.

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