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Dedicated to the memory of A.I. Morozov

Imry-Ma Phase in Systems with Defects of „Random Local Field“ and „Random Local Anisotropy“ Types (Review)

© A.A. Berzin[1], A.I. Morosov[2], A.S. Sigov[1,3,¶]

[1] MIREA — Russian Technological University,
Moscow, Russia
[2] Moscow Institute of Physics and Technology (National Research University),
Dolgoprudny, Moscow Region, Russia
[3] Moscow Technical University of Communications and Informatics,
Moscow, Russia

¶ E-mail: sigov@mirea.ru

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The conditions for the appearance in systems with defects of the „random local field“ and „random local anisotropy“ type of the Imry–Ma phases, in which the direction of the order parameter follows large-scale fluctuations of a random field or random anisotropy, are considered. It is shown that the anisotropy of the distribution of random fields of defects in the space of the order parameter can lead to the appearance of the long-range order. An attention is paid to phase diagrams arising as a result of competition between random fields of defects and anisotropy induced by defects. By the example of a system with random anisotropy (nanocrystalline ferromagnet), the dependences of the coercive field on the crystallite size are considered.

Keywords: defects of „random local field“ type, defects of „random local anisotropy“ type, phase transformations, Imry–Ma phase, phase diagrams, nanocrystalline ferromagnet.

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1. Introduction

At the turn of the last quarter of the XX century, Y. Imry and S.-k. Ma published their famous paper [1] dealing with systems having continuous symmetry of the order parameter ($O(n)$-systems, $n \geq 2$ is the number of components of the order parameter). They showed that the injection into a system of an arbitrarily weak random field, conjugate to the order parameter the correlation function of which droops at short distances, causes breakdown of the long-range order in spaces with dimension $d < 4$ even in the system ground state (the Imry–Ma theorem). The most energetically favorable is the state where the order parameter follows the large-scale static fluctuations of the random field direction. In literature it was called the Imry–Ma state (phase). The value $d_l = 4$ was called the lower critical dimension. The system's ground state at spatial dimension $d > d_l$ is the state of long-range order, and at $d < d_l$ — the Imry–Ma state. The lower critical dimension is $d_l = 2$ for Ising systems with a single-component order parameter.

Literature often uses the term „Larkin–Imry–Ma theory“. In 1970 A.I. Larkin published a paper [2] where he showed, under the framework of the Ginzburg–Landau phenomenological theory, that presence of defect-induced spatial fluctuations of coefficients preceding the terms, quadratic in the order parameter, and the gradient terms in free energy decomposition, causes disappearance of the long-range order in the location of Abrikosov vortex lines in a mixed state of a semiconductor of the second type. Both classical papers [1,2] deal with breakdown of long-range order, however, it seems unreasonable to combine them into a unified theory. Firstly, the Ginzburg–Landau phenomenological theory is true in the vicinity of the phase-transition point and its predictions cannot be directly referred to the system's ground state, which was described on the basis of a microscopic Hamiltonian in [1]. Secondly, a change in coefficients preceding the terms that are quadratic in the order parameter in the free energy decomposition does not reflect the influence of a random field, conjugate to the order parameter, since this contribution is linear in terms of the order parameter. Further analysis is based on an Imry–Ma microscopic model and the relevant theory name is used.

In their paper [1] the authors did not specify a random field source, they merely stated that it can be created by randomly frozen centers. In most of their subsequent

papers, the authors supposed that a random field is present in each node of the spin lattice. This overview analyzes the systems where a random field is created by chaotically located point defects (defects of the „random local field" type), dimensionless concentration of which c (their average quantity per one cell) is small ($c \ll 1$).

We will not dwell on a description of the transition from a paraphase to a long-range order phase in a system containing such defects, since the available advances in description of the temperature region, adjoining the phase transition point, are insignificant, and only a qualitative description of system behavior has been made so far [3,4]. Outside the region of strong fluctuations, defined by the Levanyuk–Ginzburg criterion [5], the mean field approximation holds true and the theory of quasi-isolated defects is applicable [6,7].

Mathematical modeling of systems with defects of the „random local field" type faces great challenges due to the presence of many metastable states. In particular, if the number of components of the order parameter is $n \leq d$, the presence of topological defects causes a behavior similar to spin glass behavior [8].

A phase transition from the paraphase to the Imry–Ma phase is similar to the phase transition of glass formation. The main fluctuations in the system above the temperature T_{I-M} of this transition are dynamic fluctuations of the order parameter. Below T_{I-M} there are „frozen" static fluctuations of the order parameter that follow the large-scale fluctuations of the direction of the random defect field. Value of T_{I-M} is found from the equation of the correlation radius for the order parameter of a defect-free system and the typical scale of static fluctuations in the Imry–Ma phase [9].

The most vivid and unexpected result of the study of $O(n)$-models with a random field was the prediction, based on theoretical consideration and numerical modeling, of the phenomenon of a long-range *random field induced order — RFIO*, which arises at the end temperature in a two-dimensional $X-Y$-model with collinear random fields [10,11]. It is well known that, in the absence of a random field according to Mermin–Wagner–Hohenberg theorem, the long-range order in a two-dimensional $O(n)$-model is absent at a non-zero temperature [12,13]. Later, the presence of long-range order was confirmed in a similar three-dimensional model [14–16]. It is also non-trivial, since the dimension „three" is less than the critical one and, according to the Imry–Ma theorem, the long-range order in it must be absent. A principal role in explanation of this phenomenon is played by distribution of random field directions in the space of the order parameter. As will be shown below, the Imry–Ma theorem is true only in case of an ideally isotropic distribution. Anisotropic distribution of random fields induces an effective global anisotropy in the space of the order parameter [17,18]. The occurrence of effective anisotropy disrupts the continuous symmetry of the order parameter and moves the system to another universalism class. The effective number of order parameter components decreases. In case of the $X-Y$-model, it decreases from two to one, causing the occurrence of long-range order at a non-zero temperature.

The competition of two aspects of defect influence: on the one hand — their random fields, and on the other hand — effective anisotropy induced by them, results in non-trivial phase diagrams, to the description of which our overview is largely dedicated. Moreover, the overview considers the possibility of occurrence of Imry–Ma phases in systems with a space dimension that exceeds the lower critical dimension.

As early as 1978, the Imry–Ma theorem was applied to random anisotropy systems [19,20]. They include $O(n)$-systems that contain point defects, which cause anisotropy of the „easy axis" type at the place of their location, thereat, the direction of the easy axis has a random nature (defects of the „random local anisotropy" type). Special cases of random anisotropy systems should include nanocrystalline uniaxial ferromagnets with random orientation of crystallites' crystalline lattices [21,22] and, consequently, with random orientation of easy axes, as well as liquid crystals in a porous matrix [23–25] and superfluid ^{3}He–A in aerogel [26–28].

The overview structure is as follows: the second section gives the Imry and Ma arguments for the cases of multicomponent and single-component order parameters and demonstrates the possibility of occurrence of an Imry–Ma phase in the space of a dimension that exceeds the lower critical dimension. The third section describes the anisotropy induced by anisotropic distribution of random defect fields in the space of the system order parameter. The fourth section describes the phase diagrams resulting from the competition of random defect fields and defect-induced anisotropy. The fifth section describes systems having defects of the „random local anisotropy" type, and the sixth one — an important particular case of such systems: nanocrystalline ferromagnets.

2. Imry–Ma theorem

2.1. Classical spin system

The results given in this overview are based on consideration of the classical spins of a unit length, since the latter can be included, without restriction of generality, into the corresponding interaction or field constants.

Exchange interaction energy of n-component localized spins s_i, that form a d-dimensional simple cubic lattice, in the approximation of nearest neighbors' interaction is as follows

$$W_{ex} = -\frac{1}{2} J \sum_{i,\delta} \mathbf{s}_i \mathbf{s}_{i+\delta}, \qquad (1)$$

where J is the exchange integral, summation by i is done for the entire spin lattice, and by δ — for the nearest neighbors to the given spin.

The energy of spins' interaction with random local fields of defects is equal to

$$W_{def} = -\sum_l \mathbf{s}_l \mathbf{h}_l, \tag{2}$$

summation is done for defects randomly located in lattice nodes, while distribution density of random local fields $\mathbf{h}$ in the spin space (order parameter space) has the property $\rho(\mathbf{h}) = \rho(-\mathbf{h})$, which ensures absence of a mean field in the infinite system. This overview is limited to consideration of the case of weak defect fields, when $|\mathbf{h}_l| \ll J$.

2.2. Imry–Ma arguments for $n \geq 2$

Let us reproduce the Imry–Ma arguments for the above-mentioned model. Let us consider the region of a d-dimensional space with typical linear size L expressed in units of the spin lattice constant. The number of defects in volume L^d is equal to cL^d. A typical average value of a defect field in this region, which differs from zero due to static fluctuations of the number of defects having different directions of the random field, by order of magnitude is equal to [29] $\sqrt{c\langle \mathbf{h}_l^2 \rangle / L^d}$, where angle brackets mean averaging by fields of all defects.

In the Imry–Ma phase, when the average order parameter in each such region is directed along the mean defect field, due to its interaction with the field, the system gets an energy gain. The corresponding negative addition to the ground-state energy per a spin cell by order of magnitude is

$$w_1 \approx -\left(\frac{c\langle \mathbf{h}_l^2 \rangle}{L^d} \right)^{1/2}. \tag{3}$$

However, in this case inhomogeneity of the order parameter along d directions on the typical scale L arises in the Imry–Ma phase. Since the order parameter is turning continuously, the next positive contribution to energy (per cell) occurs due to the energy of inhomogeneous exchange

$$w_2 \approx \frac{dJ}{L^2}. \tag{4}$$

It is easily seen that total energy $w_1 + w_2$ for $d < 4$ at greater values of L takes negative values. Thus, the Imry–Ma phase is energetically more favorable than the homogeneous ordered state. This proves the Imry–Ma theorem.

For weak random fields ($\langle \mathbf{h}_l^2 \rangle \ll J^2$) and $c \ll 1$ in a four-dimensional space $w_2 \gg |w_1|$ and the Imry–Ma phase does not occur.

By minimizing the total energy $w_1 + w_2$ by the parameter L, we find the optimal values of this energy and the typical inhomogeneity scale [30]:

$$w_{I-M} \approx -(4-d)\left(\frac{c\langle \mathbf{h}_l^2 \rangle}{16J^{d/2}} \right)^{\frac{2}{4-d}}, \tag{5}$$

$$L^* \approx \left(\frac{16J^2}{c\langle \mathbf{h}_l^2 \rangle} \right)^{\frac{1}{4-d}}. \tag{6}$$

Applicability of the above-mentioned estimates requires the inequation

$$c(L^*)^d \gg 1, \tag{7}$$

i.e. a fluctuation region with the optimal sizes shall contain a large number of defects. In case of weak random fields, this inequation is fulfilled automatically for $d = 2; 3$. For $d = 1$ the inequation (7) leads to the condition

$$cJ \gg \sqrt{\langle \mathbf{h}_l^2 \rangle}. \tag{8}$$

If the reverse inequation is fulfilled, i.e. with

$$cL^* < 1 \tag{9}$$

the action of defects of the „random local field“ type on the order parameter is not longer collective, and a state occurs where the order parameter in a one-dimensional spin thread turns from defect to defect so as to be turned on each defect along its random field [30]. Thereat, the typical value L is of the order c^{-1}, energy of order parameter's interaction with defects is equal to

$$w_1 \approx -c\sqrt{\langle \mathbf{h}_l^2 \rangle}, \tag{10}$$

$w_2 \ll |w_1|$ and $w_{I-M} \approx w_1$.

2.3. Imry–Ma arguments for $n = 1$

In case of Ising systems ($n = 1$), abrupt domain walls occur between the regions instead of a gradual turn of the order parameter from one region of size L to another region. Energy of such a wall per cell is of the order $2J$. Taking into account the linear concentration of domain walls ($\sim L^{-1}$), we find

$$w_2 \approx \frac{2dJ}{L}. \tag{11}$$

The defect-induced field randomly takes on value $\pm h_0$. Comparing the expressions (3) and (11), it can be concluded that with $d < d_l = 2$ the system ground state is the Imry–Ma phase. However, our reasoning did not take into account the domain wall roughness due to defects. As shown in the papers [31,32], the wall is always rough in three-dimensional systems with defects of the „random local field“ type, and the effective wall width increases as domain sizes increase. Density of domain wall surface energy in these conditions depends on L. It was later precisely demonstrated [33] that, even with consideration of the aforesaid effects, the lower critical dimension for an Ising model with defects of the „random local field“ type is equal to two.

There are opposite viewpoints on the question of existence of a long-range order in case of space dimension $d_l = 2$. The papers [34,35] gave a negative answer to this question, while the papers [36,37] have showed that a long-range order occurs in the region of weak random fields at the zero temperature. It should be noted that authors usually consider a square lattice of Ising spins, where a random field

exists on each lattice node, and its magnitude is described by a Gaussian distribution.

A simple energy consideration within the framework of a more realistic model shows [38] that with $J^2 \gg ch_0^2$ the energy of occurring domain walls is much greater than the energy gain due to interaction of the order parameter with defect fields. That is, the Imry–Ma phase does not occur, which agrees with the conclusions made in [36,37]. For final solving of this issue, the dependence of energy of a rough domain wall on domain size should be found in a two-dimensional space. If the main contribution to energy in the region of large values of L is linear by L, wall roughness will not affect the made conclusions.

The condition $d < d_l = 2$ is met only by one integer value $d = 1$, while domain walls in a one-dimensional array remain abrupt. By minimizing the total energy, we obtain values w_{I-M} and L^* for a one-dimensional Ising model with defects of the „random local field" type

$$w_{I-M} \approx -\frac{ch_0^2}{8J}, \tag{12}$$

$$L^* \approx \frac{16J^2}{ch_0^2}. \tag{13}$$

In case of weak random fields, this inequation (7) is fulfilled automatically.

2.4. Imry–Ma phase in quasi-one-dimensional systems with $n = 1$ and $d \geq d_l$

It should seem from the previous reasoning that the Imry–Ma phase cannot be the ground state for a system with a dimension exceeding the lower critical dimension. It will be shown below that this applies only to spin systems on square lattices, where exchange interaction with all the nearest neighbors is described by one exchange integral. In case of spin systems on rectangular lattices, when exchange interaction with the nearest neighbors in different directions may differ significantly, the Imry–Ma phase may also occur in case of space dimension $d \geq d_l$ [30,38]. Let us demonstrate this by the example of quasi-one-dimensional Ising systems, for which this phase may occur in a three-dimensional space [38].

It is well known that at a temperature different from the absolute zero, there is no long-range order in the chain of Ising spins. As temperature decreases in a defect-free system, the one-dimensional radius of spin correlation increases exponentially. When it reaches the critical size, weak exchange interaction of spins, pertaining to neighboring spin chains, in a quasi-one-dimensional system becomes significant. Crossover from one-dimensional behavior to d-dimensional takes place ($d \geq 2$), and a long-range order occurs in the system [39].

Presence of random defect fields may cause the occurrence of an unordered Imry–Ma state. Nature of the ground state is determined by the ration of energy of interaction between neighboring one-dimensional spin chains and energy of their interaction with defect fields.

Let us consider the following simple model.

Let the index m number the parallel spin chains that form a $(d-1)$-dimensional square lattice ($d \geq 2$) in a perpendicular cut, while the index i — spins along the given chain. Then the spin interaction energy is as follows

$$W_{ex} = -J_{\parallel} \sum_{i,m} \sigma_{i,m}\sigma_{i+1,m} - \frac{1}{2} J_{\perp} \sum_{i,m,\delta} \sigma_{i,m}\sigma_{i,m+\delta}, \tag{14}$$

summation by i and m is done for the whole spin lattice, and by δ — for the neighboring chains nearest to the given one. The exchange integral that describes the interaction of neighboring spins pertaining one chain, $J_{\parallel} > 0$, is much larger than the one for neighboring spins pertaining to different chains, $J_{\perp} > 0$. Energy of spins' interaction with random local fields of defects is described by formula (2), where the index l sets a pair of indices i_l, m_l.

Temperature of crossover from one-dimensional to d-dimensional behavior and occurrence of a long-range order in a mean field approximation is found based on the condition of transformation of spin system susceptibility into infinity [39] or based on the conditions of equality of temperature and energy of interaction of a correlated spin region, the size of which is equal to the one-dimensional correlation radius $r_{\parallel}$, with a molecular field

$$T \approx zJ_{\perp} r_{\parallel}(T), \tag{15}$$

where z is the number of spin chains nearest to the given one. In our model $z = 2(d-1)$. According to [39],

$$r_{\parallel} = \frac{1}{2} \exp\left(\frac{2J_{\parallel}}{T}\right), \tag{16}$$

which makes it possible to obtain an expression for the temperature of ferromagnetic transition in a defect-free system [39]:

$$T_c \approx \frac{2J_{\parallel}}{\ln\left(\frac{4J_{\parallel}}{zJ_{\perp}}\right)}. \tag{17}$$

A correction to the ground state energy of non-interacting defect-free spin chains owing to interaction between the chains and occurrence of a d-dimensional long-range order per cell is

$$w_d = -\frac{zJ_{\perp}}{2}. \tag{18}$$

Defects of the „random local field" type at $2J_{\parallel} > h_0$ do not change the ordered state energy owing to a random sign of the defect field. If an Imry–Ma phase occurs in the system, the correlation between spins that pertain to neighboring chains becomes disrupted, and a correction to the ground state energy of non-interacting defect-free spin chains owing to interaction with random fields is described by the formula (12) with $J \equiv J_{\parallel}$.

Occurrence of an Imry–Ma phase requires that

$$|w_{I-M}| > |w_d|, \tag{19}$$

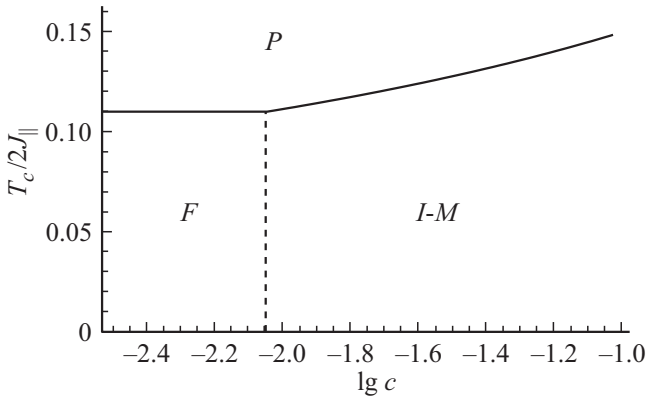

Figure 1. Phase diagram for a quasi-one-dimensional Ising model with defects of the „random local field“ type [38] with $z = 4$, $h_0^2/J_\parallel^2 = 10^{-1}$ and $J_\perp/J_\parallel = 10^{-4}$: P is the paramagnetic phase, F is the ferromagnetic phase, $I-M$ is the Imry–Ma phase.

from where we get a condition for defect concentration [38]:

$$c > \frac{4z J_\perp J_\parallel}{h_0^2}. \tag{20}$$

For $c \sim 10^{-2}$ and $h_0^2/J_\parallel^2 \sim 10^{-1}$ this gives the condition $J_\perp/J_\parallel \lesssim 10^{-4}$. In case of such a weak exchange interaction, dipole-dipole interaction between spins can become essential. For occurrence of an Imry–Ma phase, it values shall also not exceed $10^{-4}J_\parallel$.

Temperature T^* of occurrence of an Imry–Ma phase is found from the condition $L^* = r_\parallel$ [9] and equal to

$$T^* \approx \frac{2J_\parallel}{\ln\left(\frac{32J_\parallel^2}{ch_0^2}\right)}. \tag{21}$$

It is easy to see that the condition (19) is virtually equivalent to the condition $T^* > T_c$.

A phase diagram for a quasi-one-dimensional Ising model with defects of the „random local field“ type in variables „temperature — defect concentration“ [38] is shown in Fig. 1.

3. Anisotropy induced by random defect fields

3.1. Approximation quadratic in defect field. Case $2 < d < 4$

Let us show that anisotropic distribution of random fields induces an effective global anisotropy in the space of the order parameter.

Let us consider the action of a separate defect on a homogeneous ferromagnetic state with an order parameter of $\mathbf{s}_0$. Let us go to the continuous distribution of the order parameter $\mathbf{s}(\mathbf{r})$, designating its perpendicular $\mathbf{s}_0$ component $\mathbf{s}^\perp(\mathbf{r})$. Let $|\mathbf{s}_0| = 1 \gg |\mathbf{s}^\perp(\mathbf{r})|$. All distances are made „dimensionless“, by dividing them by the spin cell size. Then, in an approximation quadratic in $\mathbf{s}^\perp(\mathbf{r})$, the expression (1) takes the form of [5]:

$$W_{ex} = \frac{J}{2}\int d^d\mathbf{r}\,\frac{\partial \mathbf{s}^\perp}{\partial x_i}\,\frac{\partial \mathbf{s}^\perp}{\partial x_i}. \tag{22}$$

Energy of interaction of a random field $\mathbf{h}(\mathbf{r})$ with the order parameter $\mathbf{s}(\mathbf{r})$ has the following form in continuous representation

$$W_{def} = -\int d^d\mathbf{r}\,\mathbf{h}(\mathbf{r})\mathbf{s}(\mathbf{r}), \tag{23}$$

where

$$\mathbf{h}(\mathbf{r}) = \sum_l \mathbf{h}_l\,\delta(\mathbf{r} - \mathbf{r}_l). \tag{24}$$

Energy of random field's interaction with the longitudinal component of the order parameter is equal to zero owing to the ratio $\rho(\mathbf{h}) = \rho(-\mathbf{h})$. For simplicity, we will neglect the system's longitudinal susceptibility in the region of low temperatures, being much lower than the magnetic ordering temperature. Therefore W_{def} can be written as

$$W_{def} = -\int d^d\mathbf{r}\,\mathbf{h}^\perp(\mathbf{r})\mathbf{s}^\perp(\mathbf{r}), \tag{25}$$

where

$$\mathbf{h}^\perp(\mathbf{r}) = \sum_l [\mathbf{h}_l - \mathbf{s}_0(\mathbf{s}_0\mathbf{h}_l)]\delta(\mathbf{r} - \mathbf{r}_l) \tag{26}$$

is the component of the random field perpendicular to $\mathbf{s}_0$. It leads to a local deviation of the order parameter and occurrence of a non-zero component $\mathbf{s}^\perp(\mathbf{r})$. As a result, there is a negative addition to the ground state energy proportional to $(\mathbf{h}_l^\perp)^2$. It is maximal in modulus when the direction $\mathbf{s}_0$ is perpendicular to the local impurity field.

In the particular case of anisotropic distribution of random field directions, when all $\mathbf{h}_l$ are collinear, it is energetically favorable for the order parameter to orient perpendicularly to this direction [17]. Thus, in case of a $X-Y$-model ($n = 2$) there is anisotropy of the „easy axis“ type, while in case of a Heisenberg model ($n = 3$) — anisotropy of the „easy plate“ type. In case of a complanar distribution of random field directions, an easy axis, perpendicular to the said plane, occurs in the order parameter space in the Heisenberg model. In case of a more common anisotropic distribution of random field directions, it is favorable for the order parameter to orient perpendicularly to the predominant direction of random fields. Owing to the condition of parity of the function $\rho(\mathbf{h})$, the predominant direction of random fields should be determined by considering only the hemisphere of their directions in a n-dimensional space of the order parameter.

The Green function for the set problem is well known [40]. It assigns a relation between Fourier components of the order parameter and the random field

$$\mathbf{s}^\perp(\mathbf{k}) = \chi^\perp(\mathbf{k})\mathbf{h}^\perp(\mathbf{k}), \tag{27}$$

where

$$\chi^{\perp}(\mathbf{k}) = (Jb^2k^2)^{-1}, \tag{28}$$

$$\mathbf{h}^{\perp}(\mathbf{k}) = \frac{1}{N}\int d^d\mathbf{r}\,\mathbf{h}^{\perp}(\mathbf{r})\exp(-i\mathbf{kr})$$

$$= \frac{1}{N}\sum_l [\mathbf{h}_l - \mathbf{s}_0(\mathbf{s}_0\mathbf{h}_l)]\exp(-i\mathbf{kr}_l), \tag{29}$$

N is the number of elementary cells. Therefore,

$$\mathbf{s}^{\perp}(\mathbf{r}) = \frac{1}{N}\sum_{\mathbf{k}}\chi^{\perp}(\mathbf{k})\sum_l [\mathbf{h}_l - \mathbf{s}_0(\mathbf{s}_0\mathbf{h}_l)]\exp[i\mathbf{k}(\mathbf{r}-\mathbf{r}_l)], \tag{30}$$

summation by $\mathbf{k}$ is done for the Brillouin zone.

Substitution of this expression into formulas (22) and (25) gives the following effective anisotropy energy per cell [41]:

$$w_{an} = \frac{c\tilde{\chi}^{\perp}}{2}\left[s_{01}^2\langle h_{l1}^2\rangle + s_{02}^2\langle h_{l2}^2\rangle + \ldots + s_{0n}^2\langle h_{ln}^2\rangle\right], \tag{31}$$

where h_{lj} and $\mathbf{s}_{0j}$ are j-e components of field $\mathbf{h}_l$ and order parameter s_0, respectively, and

$$\tilde{\chi}^{\perp} = \int \frac{d^d\mathbf{k}}{(2\pi)^d}\chi^{\perp}(\mathbf{k}). \tag{32}$$

Value of $\tilde{\chi}^{\perp}$ in a space of dimension $2 < d < 4$ does not have peculiarities in case of $\mathbf{k} = 0$.

In particular, in case of collinear orientation of random fields, anisotropy energy is as follows

$$w_{an} = \frac{1}{2}c\tilde{\chi}^{\perp}\langle\mathbf{h}_l^2\rangle\cos^2\varphi \equiv \frac{1}{2}K\cos^2\varphi, \tag{33}$$

where φ is the angle between the order parameter vector and axis of „hard magnetization“, to which random impurity fields are collinear, while K is the anisotropy constant.

In case of a complanar and isotropic (in the assigned plane) distribution of random fields in a Heisenberg model, volumetric density of anisotropy energy is equal to

$$w_{an} = -\frac{1}{4}c\tilde{\chi}^{\perp}\langle\mathbf{h}_l^2\rangle\cos^2\varphi \equiv -\frac{1}{2}K\cos^2\varphi, \tag{34}$$

where φ is the angle between the order parameter vector and the normal line to the plane where random fields lie.

For $d = 3$ the value is $\tilde{\chi}^{\perp} \sim 0.2/J$. Thus, the effective anisotropy constant is a value of the order $K \sim 0.1c\langle\mathbf{h}_l^2\rangle/J$.

In case of a more common ellipsoidal distribution of random fields, we choose the following value as an effective anisotropy constant K

$$K = 2(w_{an}^{\max} - w_{an}^{\min}), \tag{35}$$

where $w_{an}^{\max}$ and $w_{an}^{\min}$ are the maximum and minimum values of the expression (31) as functions of the direction of vector $\mathbf{s}_0$.

For a $X-Y$-model with $|\mathbf{h}_l| = \text{const}$,

$$\rho(\mathbf{h}) = A[h_x^2 + (1+\varepsilon)h_y^2], \tag{36}$$

where the constant A is found from the normalization condition, $\varepsilon > -1$, while the value of K, calculated using the formula (35), is equal to

$$K = \frac{|\varepsilon|\tilde{\chi}^{\perp}c\langle\mathbf{h}_l^2\rangle}{2(2+\varepsilon)}. \tag{37}$$

With any sign of ε, defects induce anisotropy of the „easy axis“ type, direction of which is perpendicular to the predominant direction of random fields.

For a Heisenberg model with the distribution $|\mathbf{h}_l| = \text{const}$,

$$\rho(\mathbf{h}) = A[h_x^2 + h_y^2 + (1+\varepsilon)h_z^2], \tag{38}$$

the value of K is equal to

$$K = \frac{2|\varepsilon|\tilde{\chi}^{\perp}c\langle\mathbf{h}_l^2\rangle}{5(3+\varepsilon)}. \tag{39}$$

In case of $-1 < \varepsilon < 0$, an easy axis z occurs in the system, and in case of $\varepsilon > 0$ — an easy plane xy.

3.2. Approximation quadratic in defect field. Case $d = 2$

A peculiarity of two-dimensional models is the absence of a long-range order in a pure system at the end temperature. This is evidenced, in particular, by the logarithmic divergence of value $\tilde{\chi}^{\perp}$ while the lower limit of integration in modulus $\mathbf{k}$ tends to the value $k = 0$. Therefore, the problem should be solved in a self-consistent way, making an assumption of the presence of global anisotropy induced by random fields [18], the energy of which is written as

$$W_{an} = \frac{1}{2}K\int d^2\mathbf{r}\left(s_{01}(\mathbf{r})\right)^2. \tag{40}$$

Since the random field makes the order parameter deviate from the easy direction to the hard one, then $\chi^{\perp}(\mathbf{k})$ is as follows

$$\chi^{\perp}(\mathbf{k}) = (Jk^2 + K)^{-1}. \tag{41}$$

The constant K cuts the divergence of the integral $\tilde{\chi}^{\perp}$ in case of small $\mathbf{k}$, which make the main contribution to $\tilde{\chi}^{\perp}$ with $d = 2$. As a result

$$\tilde{\chi}^{\perp} = \frac{1}{4\pi J}\ln\frac{4\pi J}{K}. \tag{42}$$

The value of K is found an iterative solution of a self-coupling equation that arises after substitution of the expression (42) in formulas (33) or (34). After the first iteration for the case of collinear random fields, we have [41]:

$$K = \frac{c\langle\mathbf{h}_l^2\rangle}{4\pi J}\ln\frac{16\pi^2J^2}{c\langle\mathbf{h}_l^2\rangle}, \tag{43}$$

while for the case of a complanar and an isotropic (in the assigned plane) distribution of random fields with $n = 3$, we identically obtain

$$K = \frac{c\langle\mathbf{h}_l^2\rangle}{8\pi J}\ln\frac{32\pi^2J^2}{c\langle\mathbf{h}_l^2\rangle}. \tag{44}$$

3.3. Fourth-order approximation by defect field

Possibly, the following equation is met in case of anisotropic distribution of directions of random local defect fields in a n-dimensional space of a vectorial order parameter

$$\langle h_{l1}^2 \rangle = \langle h_{l2}^2 \rangle = \cdots = \langle h_{ln}^2 \rangle. \quad (45)$$

In this case, an effective anisotropy quadratic in $\mathbf{h}^{\perp}(\mathbf{r})$ does not arise in the system. This distribution of random defect fields corresponds to a situation when defect fields with equal probability are collinear to n mutually perpendicular directions in the order parameter space, which for consideration convenience were chosen as the axes of the Cartesian coordinate system. Then anisotropy energy per cell is as follows

$$w_{an} = K \sum_{j=1}^{n} s_{0j}^4. \quad (46)$$

In order to obtain the constants of anisotropy, proportional to the fourth powers of random fields' components, the following powers of exchange interaction energy decomposition (22) by the powers $\mathbf{s}^{\perp}(\mathbf{r})$ [41] must be taken into account.

It results in the following order estimate of the value for the effective anisotropy constant K in a three-dimensional space

$$K \approx 10^{-3} \frac{c\langle \mathbf{h}_l^4 \rangle}{J^3}. \quad (47)$$

Since $K > 0$, the vector $\mathbf{s}_0$ in an equilibrium state is directed along one of the main diagonal lines of the Cartesian coordinate system in the order parameter space.

As compared to the case of collinear orientation of random fields, the effective anisotropy constant contains a small parameter $\langle \mathbf{h}_l^2 \rangle / J^2$, which is natural since energy decomposition by even degrees of a random field takes place exactly for this dimensionless parameter.

Quantity of order parameter components is not essential, because anisotropy of the fourth and higher orders by $\mathbf{h}_l$ induces the formation of easy axes, but not easy planes.

In a two-dimensional coordinate space, the problem should be solved in a self-coordinated way, as in the previous section. As a result of the first iteration, the expression for the anisotropy constant will be as follows [41]:

$$K \approx \frac{c\langle \mathbf{h}_l^4 \rangle}{J^3} \ln \frac{J^4}{c\langle \mathbf{h}_l^4 \rangle}. \quad (48)$$

4. Phase diagrams

4.1. Critical value of the constant for anisotropy of the „easy axis“ type and the Imry–Ma theorem

Evidently, the weak defect-induced anisotropy studied in the previous section will not affect the phase diagram of single-component systems which already have strong anisotropy, but can, by removing the degeneration, significantly affect the behavior of $O(n)$-models.

The paper [29] has showed that the presence of weak crystalline anisotropy of the „easy axis“ type causes the following: an Imry–Ma phase occurs in a space of dimension $2 \le d < 4$ only if concentration of defects of the „random local field“ type exceeds the critical value. Indeed, homogeneous ordering of spins of a collinearly easy axis reduces the energy in relation to the value that corresponds to the homogeneous state of an isotropic system, by $K/2$ per cell. In the heterogeneous Imry–Ma state, the order parameter deviates from the easy axis and no such energy gain takes place. Therefore, the Imry–Ma phase will be energetically favorable if

$$|w_{I-M}| > K/2, \quad (49)$$

where w_{I-M} is given by the expression (5).

The critical value of the anisotropy constant, that satisfies the conversion of this inequation into an equation, by order of magnitude is [29]:

$$K_{\mathrm{cr}} \approx J \left[\frac{c\langle \mathbf{h}_l^2 \rangle}{J^2} \right]^{\frac{2}{4-d}}. \quad (50)$$

If $K > K_{\mathrm{cr}}$, then a long-range order arises in the system, while in case of $K < K_{\mathrm{cr}}$ the Imry–Ma phase satisfies the ground state. This rule remains unchanged also when anisotropy is not crystallographic in nature, but is created by the defects themselves.

It is well known [3] that a nonmagnetic substitutional impurity or a vacancy in a two-sublattice collinear antiferromagnet, being in an external magnetic field, is a defect of the „random local field“ type. The local field value is directly proportional to density of the applied magnetic field. The comparison of $|w_{I-M}|$ with the energy of crystalline anisotropy shows that in the collinear phase the inequation (49) is not fulfilled [42]. The role of crystalline anisotropy in the spin-flop phase is played by an external field that orients the magnetizations of antiferromagnet sublattices so that the resultant magnetic moment, arising due to their skewing, is directed along the field. And energy of this effective anisotropy in the given phase exceeds $|w_{I-M}|$. Thus, the Imry–Ma phase is not implemented in an antiferromagnet with an identical exchange interaction with all the nearest neighbors. It may arise in the collinear phase of a quasi-one-dimensional two-sublattice collinear antiferromagnet in the case of fulfillment of the conditions formulated in Section 2.4.

It is seen from the previous overview section that the effective anisotropy constant, created by anisotropic distribution of defect fields, is proportional to defect concentration c. At the same time, value of K_{cr} in a space with dimension $2 < d < 4$ contains a higher power of c.

In particular, in case of a three-dimensional coordinate space $K_{\mathrm{cr}} \propto c^2$. From this it follows that effective anisotropy within the limit $c \to 0$, arising in any order by $\mathbf{h}_l$, will

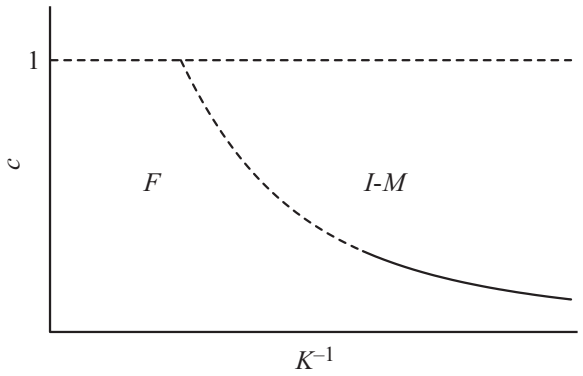

Figure 2. Phase diagram for a three-dimensional system [41] in variables „defect concentration c — constant for effective anisotropy of the „easy axis“ type K“ : F — ferromagnetic phase, $I-M$ — unordered Imry–Ma phase.

exceed the critical value. In case of $d = 2$, quantity $K \propto -c \ln c$, that is this quantity in the small concentration region also exceeds the critical value $K_{\text{cr}} \propto c$. Thus, the Imry and Ma theorem in spaces of dimension $2 \leq d < 4$ is not true at any arbitrarily weak effective anisotropy of the „easy axis“ type, induced by random local defect fields. In systems having a dimension $d < 2$ within small concentrations $K < K_{\text{cr}}$ and the Imry and Ma theorem is true.

Comparing the expressions (37), (39) and (50), one can make sure that in the case of $d = 3$ and strongly anisotropic distributions of random fields, which cause anisotropy of the „easy axis“ type, the Imry–Ma state is not implemented in the whole possible range of defect concentrations $c < 1$.

The condition $K < K_{\text{cr}}$ for weakly anisotropic distributions of random fields gives a lower bound for defect concentration, at which an unordered Imry–Ma state is observed. For instance, with $d = 3$ the following takes place for a distribution of random defect fields, set by the formula (37)

$$c > 0.1\varepsilon \frac{J^2}{\langle \mathbf{h}_l^2 \rangle}. \tag{51}$$

With $J^2/\langle \mathbf{h}_l^2 \rangle \sim 100$ and $\varepsilon \sim 10^{-3}$ we have $c > 0.01$. A typical phase diagram of the system ground state [41] is given in Fig. 2.

Concentration dependences K and K_{cr} in a two-dimensional space differ by a logarithmic dependence, therefore, given the difference of the numerical coefficients in these dependences, the Imry–Ma phase can be also observed in the case of strongly anisotropic distributions of random fields [9].

The paper [43], in a theoretical study of a $O(n)$-system with weak uniaxial anisotropy, found that the injection of impurities of the „random local field“ type with fields, collinear to the easy axis, causes (as the impurity concentration increases) a transition to the phase where the order parameter is perpendicular to the easy axis. The author interpreted this as a defect-induced spin-flop transition. Taking into account the aforesaid consideration, it should be more correctly classified as an orientational transition. Indeed, introduction of impurities reduces the anisotropy constant value, and its sign changes in case of a critical impurity concentration, that is, anisotropy of the „easy plane“ type arises.

4.2. Anisotropy of the „easy plane“ type

When defects cause effective anisotropy of the „easy plane“ type, the situation is not so simple. If anisotropy is weak ($K < K_{\text{cr}}$), the Imry–Ma phase satisfies the ground state. Otherwise ($K > K_{\text{cr}}$), the issue of the system ground state should be solved by projecting all random fields onto an easy hyperplane of the dimension m ($n > m \geq 2$) in the order parameter space and by considering the problem on the said hyperplane. The operation should be repeated if anisotropy of the „easy plane“ type with $K > K_{\text{cr}}$ arises in it. As a result, we get one of the three possible cases:

– Projections of random fields onto the easy plane are equal to zero. Thereat, system's behavior is identical to behavior of a pure system with a number of order parameter components that corresponds to the hyperplane dimension. An inhomogeneous Imry–Ma state does not arise, though random fields induce the occurrence of order parameter components that are perpendicular to the easy plane.

– Anisotropy of the „easy axis“ type takes place in the easiest plane. Then the problem reduces to a problem with anisotropy of the „easy axis“ type, but with a number of order parameter components equal to m.

– The distribution of projections of random local defect fields onto the easy plane meets the condition $K < K_{\text{cr}}$. The system ground state in this case is the Imry–Ma phase.

4.3. Temperature phase diagrams

Temperature of occurrence of the Imry–Ma phase in a one-dimensional Ising model is given by the formula (21), where $J_{\parallel}$ should be replaced by J. This is the sole dimension for a system with $d < d_l$.

Let us consider $O(n)$-systems ($n \geq 2$) in spaces with dimension $d < d_l$, that is, in one-dimensional, two-dimensional and three-dimensional spaces. In a three-dimensional space, in the absence of defects, a long-range order in $O(n)$-systems arises at the end temperature T_c. As already noted, temperature of transition to the Imry–Ma phase (T_{I-M}) is found from the equation of the correlation radius for the order parameter of a defect-free system r_c and the typical scale L^* of static fluctuations in the Imry–Ma phase [9]:

$$r_c(T_{I-M}) = L^*. \tag{52}$$

Since $L^* \gg 1$, and r_c diverges near the point of second-order phase transition from the paraphase into the ordered phase, then for $d = 3$ we have $T_{I-M} \sim T_c$. That is, a transition from the paraphase into the phases shown in Fig. 2 occurs at $T \sim T_c$.

A similar situation takes place in a two-dimensional $X-Y$-model. It is well known that a phase transition takes place in it at the end temperature $T_{BKT} = \pi J/2$: from the paraphase with an exponentially drooping correlation function of the order parameter into the Berezinskii–Kosterlitz–Thouless (BKT) phase with a power mode of correlation function drooping [44–46]. According to [47], the correlation radius behaves as

$$r_c = \exp(b\tau^{-1/2}), \qquad (53)$$

where $b = \text{const}$, $\tau = (T - T_{BKT})/T_{\text{BKT}}$, that is, it diverges near the transition temperature. Consequently, similarly to a three-dimensional system, $T_{I-M} \sim T_{BKT}$.

The formation of arbitrarily weak anisotropy in a system leads to the occurrence of a long-range order [48] and the BKT phase does not arise. An estimate for the transition temperature T_c can be found from simple energy considerations [9]. For this, we equate (by absolute value) T_c and anisotropy energy $\sim \pi K r_c^2$ of a correlated round region having a radius equal to correlation radius r_c. The result is $T_c \sim T_{BKT}$. Since real crystalline systems always contain weak anisotropy induced by the crystalline lattice symmetry, experimental observation of the BKT phase in such systems is hardly possible.

When anisotropy is created by random defect fields, the system phase diagram is richer: if anisotropy is more than the critical one, the phase with long-range order arises, and if it is less than the critical one, then the Imry–Ma phase.

Let us consider a particular case of anisotropic distribution of random fields' directions, where all $\mathbf{h}_l$ are collinear. As already note, an easy axis occurs in the system; it is perpendicular to the random fields' direction, while the anisotropy constant value is given by the formula (43). The condition $K > K_{\text{cr}}$ is fulfilled in the region of concentrations $c < c_{\text{cr}} = 5.5 \cdot 10^{-2}$. The order parameter in the arising ordered phase is collinear to the easy axis, its large-scale static fluctuations are suppressed, there are only local deviations of spins near defects.

With greater concentrations, when $K < K_{\text{cr}}$, the Imry–Ma phase with a long-range order is implemented in the system. The reasons for occurrence of this unique phase are outlined in the paper [49]. A multitude of random fields' directions in the $X-Y$-model can be set by points on a unit circumference in the order parameter space. In case of fields collinear to the ξ axis of the Cartesian orthogonal coordinate system (ξ, η) in the given space, a multitude of random fields' directions is set by points A and B in Fig. 3. Upon transition from the region having a typical size L^* and one field direction to a neighboring region with the opposite field direction, the order parameter makes a 180° turn. Thereat, the order parameter direction changes either along the ACB arc, or along the ADB arc. Energy of interaction with random fields does not depend on this choice. Energy of inhomogeneous exchange will be admittedly lesser if the order parameter in the whole spin lattice turns along the same arc, e.g., ACB. Thereat, a mean value of the vectorial order parameter, parallel to the η axis, will occur in the system. It is far from saturation even in the ground state, since there is a second component of the order parameter that changes its sign from region to region. We believe that this phase is the closest to the one considered the papers [10,11] where the authors introduced a random field into each cell ($c = 1$) and demonstrated the presence of a long-range random field induced order.

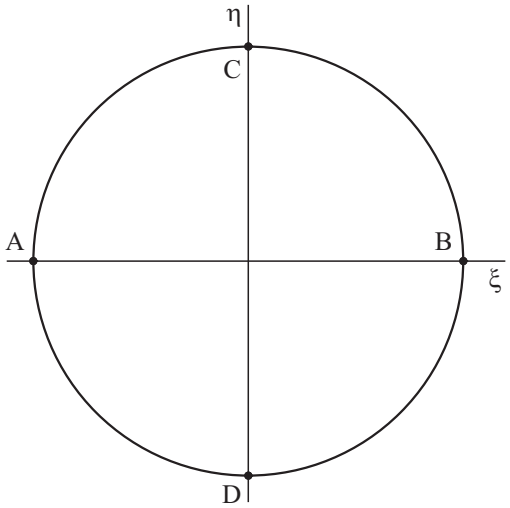

Figure 3. Space of directions of the order parameter for the $X-Y$-model.

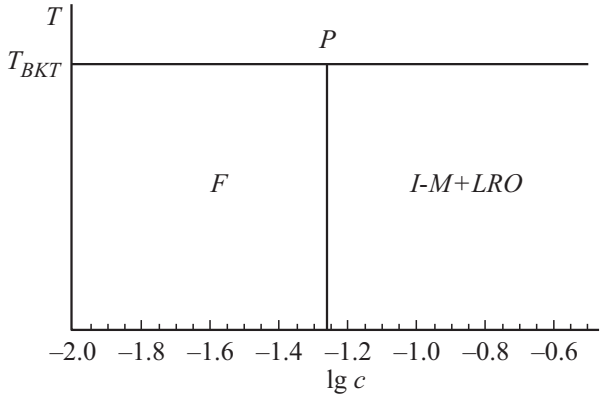

Figure 4. Phase diagram of a two-dimensional $X-Y$-model with collinear directions of random defect fields [9] with $J^2/\langle \mathbf{h}_l^2 \rangle = 100$: P — paramagnetic phase, F — ferromagnetic phase, $I-M+LRO$ — Imry–Ma phase with a long-range order.

The phase diagram of a two-dimensional $X-Y$-model with defects of the „random local field“ type [9] is given in Fig. 4.

In the remaining unconsidered cases: $d = 1$; $n \geq 2$ and $d = 2$; $n \geq 3$, the long-range order in a defect-free isotropic system arises only at the absolute zero temperature. The correlation radius diverges as temperature approaches the absolute zero (see, for instance, the data on the two-dimensional Heisenberg model [47],

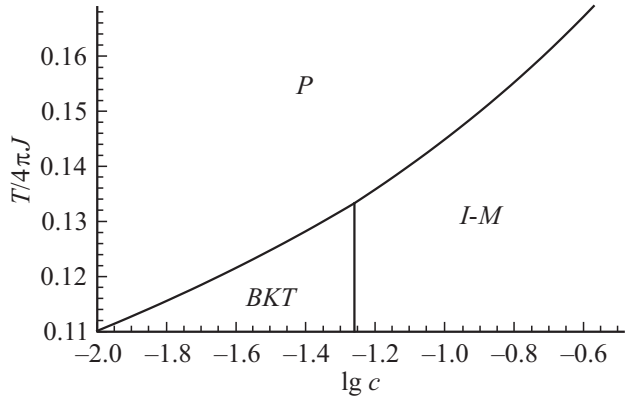

Figure 5. Phase diagram of a two-dimensional Heisenberg model with anisotropy of the „easy plane“ type, induced by defects of the „random local field“type [9], with $J^2/\langle \mathbf{h}_l^2 \rangle = 100$: P — paramagnetic phase, BKT — BKT phase, $I-M$ — unordered Imry–Ma phase.

one-dimensional $X-Y$ model [50,51] and one-dimensional Heisenberg model [52,39]).

A defect-free two-dimensional Heisenberg model with weak anisotropy was considered in the paper [53]. In case of anisotropy of the „easy axis“ type, the system goes to the class of Ising models, and a long-range order arises in it at temperature T_c equal to

$$T_c \approx \frac{4\pi J}{\ln\left(\frac{J}{K}\right)}. \tag{54}$$

The expression (54) is found, similarly to the case of a two-dimensional $X-Y$-model, from the ratio $T_c \sim \pi K r_c^2$. Correlation radius for a two-dimensional Heisenberg model is given by the formula [47]:

$$r_c = \exp\left(\frac{2\pi J}{T}\right). \tag{55}$$

If weak anisotropy of the „easy plane“ type arises in a two-dimensional Heisenberg model, then a crossover to a two-component order parameter and a phase transition to the BKT phase [53] take place at temperature T_c, which is set by the formula (54).

When anisotropy is created by defects of the „random local field“ type, the recommendations outlined in the two previous sections should be followed. Value of temperature of transition to the Imry–Ma phase determined by the equation (52) is equal to

$$T_{I-M} \approx \frac{2\pi J}{\ln L^*} \approx \frac{4\pi J}{\ln\left(\frac{J^2}{c\langle \mathbf{h}_l^2 \rangle}\right)} \approx \frac{4\pi J}{\ln\left(\frac{J}{K_{\rm cr}}\right)}. \tag{56}$$

It should be noted that the condition $K > K_{\rm cr}$ is equivalent to the condition $T_c > T_{I-M}$.

In a particular case, when all defect fields are collinear to the z axis of the Cartesian orthogonal coordinate system in the spin space, an easy plane xy arises in it. A crossover to a two-component order parameter takes place in the region of values $K > K_{\rm cr}$ at $T \approx T_c$. Since projections of random fields onto an easy plane are equal to zero, which makes the arising system equivalent to a pure $X-Y$-model, and $T_c \ll J$, then the crossover is accompanied by a phase transition to the BKT phase. With $K < K_{\rm cr}$, the Imry–Ma phase is implemented. The phase diagram of a two-dimensional Heisenberg model with defects of the „random local field“ type and anisotropy of the „easy plane“ type [9] is given in Fig. 5.

The long-range order in one-dimensional $O(n)$-systems at temperatures other than the absolute zero is not present [5], while a transition to the Imry–Ma phase occurs at the end temperature T_{I-M} determined by the ratio (52). The paper [30] generalized the calculation made in the paper [52] to obtain an expression for the correlation radius for a one-dimensional system with a n-component order parameter

$$r_c = \frac{2J}{(n-1)T}. \tag{57}$$

As a result, we obtain

$$T_{I-M} \approx \begin{cases} \dfrac{1}{n-1}\left(\dfrac{cJ\langle \mathbf{h}_l^2 \rangle}{2}\right)^{1/3}, \text{ if} cJ \gg \sqrt{\langle \mathbf{h}_l^2 \rangle}, \\ \dfrac{2cJ}{n-1}, \text{ if} cJ \ll \sqrt{\langle \mathbf{h}_l^2 \rangle}. \end{cases} \tag{58}$$

The Imry–Ma phase with a long-range order must be observed in the particular case of collinear random fields at the absolute zero temperature.

5. $O(n)$-systems with defects of the „random local anisotropy“ type

5.1. Arguments similar to those given by Imry and Ma

Let us consider a $O(n)$-model that contains point defects of the „random local anisotropy“ type. Energy of spins' interaction with such defects is equal to

$$W_{imp} = -K_0 \sum_l (\mathbf{s}_l \mathbf{n}_l)^2, \tag{59}$$

where $K_0 > 0$ is the constant of defect-induced anisotropy, summation is done for defects randomly located in lattice nodes, while $\mathbf{n}_l$ is the unit vector that sets a direction for the random easy axis.

Let us consider a case of chaotic distribution of local easy anisotropy axes in the order parameter space. Due to static fluctuations in the space region of a system having a typical linear size L, impurities with a certain direction of easy axes are predominant and mean anisotropy with the constant $K_L \sim K_0\sqrt{c/L^d}$ arises. When the vector of the order parameter follows the static spatial fluctuations of the

easy axis direction, there is an energy gain as compared to the homogeneous state. Per cell, its value is

$$w_1 \approx -K_L \approx -K_0\left(\frac{c}{L^d}\right)^{1/2}. \qquad (60)$$

Comparing it to the energy of inhomogeneous exchange (4), we can easily see that the Imry–Ma phase in case of $d < 4$ at large values of L is energetically more favorable that a homogeneous ordered state.

The optimal scale of inhomogeneity L^*, satisfying the minimum total energy, and an addition to the homogeneous state energy per cell are respectively equal to

$$L^* \approx \left(\frac{16J^2}{cK_0^2}\right)^{\frac{1}{4-d}}, \qquad (61)$$

$$w_{I-M} \approx -(4-d)\left(\frac{cK_0^2}{16J^{d/2}}\right)^{\frac{2}{4-d}}. \qquad (62)$$

Formulas for defects of the „random local anisotropy“ type are obtained from the formulas for defects of the „random local field“ type by replacing $\langle \mathbf{h}_l^2 \rangle$ with K_0^2. In particular, the critical anisotropy constant is equal to [54]

$$K_{\text{cr}} \approx J\left[\frac{cK_0^2}{J^2}\right]^{\frac{2}{4-d}}. \qquad (63)$$

5.2. Anisotropic distribution of easy axes

As distinct from defects of the „random local field“ type, actual occurrence of global anisotropy in case of anisotropic distribution of easy axes in the order parameter space seems obvious. Anisotropy occurs in an approximation which is linear in K_0. By replacing the components of the random field vector in the formulas (36) and (38) with the components of vector n, we obtain an anisotropy constant described by the formulas (37) and (39), where $\tilde{\chi}^{\perp}\langle \mathbf{h}_l^2 \rangle$ should be replaced with K_0.

When global anisotropy, linear in the constant K_0, does not arise (e.g., when defects' easy axes are equally probably parallel to the axes of an orthogonal Cartesian coordinate system), the following terms of energy decomposition by powers of the parameter K_0/J can be taken into account and cubic anisotropy can be obtained. Its energy per cell is equal to [55,56]

$$w_{an} = \frac{cK_0^2\tilde{\chi}^{\perp}}{2}\langle(\mathbf{s}_0\mathbf{n}_l)^4\rangle = \frac{cK_0^2\tilde{\chi}^{\perp}}{2n}\sum_{j=1}^{n} s_{0j}^4, \qquad (64)$$

the angle brackets mean averaging by easy axes of all defects.

Further comparison of the arising anisotropy with the critical value is completely identical to the case of defects of the „random local field“ type. The condition of Imry–Ma phase existence in case of global anisotropy of the „easy axis“ type gives the following limitation on the asymmetry

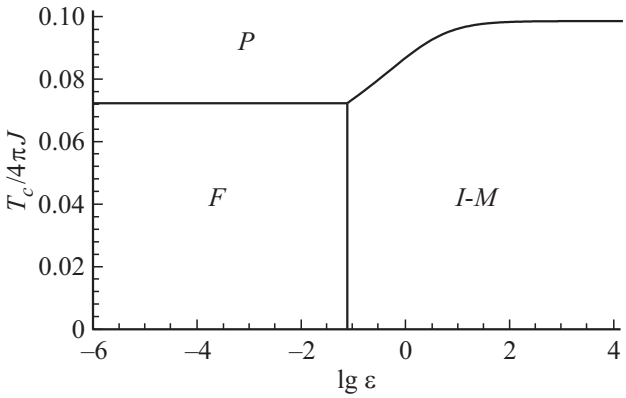

Figure 6. Phase diagram of a two-dimensional Heisenberg model with defect-induced (defect of the „random local anisotropy“ type) global anisotropy of the „easy axis“ type [58] in the variables „temperature–asymmetry“ with $K_0/J = 10^{-2}$ and $c = 10^{-2}$: P — paramagnetic phase, F — ferromagnetic phase, $I-M$ — Imry–Ma phase.

degree of easy axes distribution in the order parameter space [54]:

$$\varepsilon < (1 \div 10)\, c^{\frac{d-2}{4-d}}\left(\frac{K_0}{J}\right)^{\frac{d}{4-d}}. \qquad (64)$$

For $d = 3$ we obtain, in case of the values $c \sim 10^{-2}$ and $K_0/J \sim 10^{-2}$, a region of values $\varepsilon < 10^{-7}$ (for comparison, in case of defects of the „random local field“ type, the limitation $\varepsilon < 10^{-3}$ has been obtained in Section 4.1 with $c \sim 10^{-2}$ and $h_l \sim 0.1J$). In a real sample, it is difficult to create an isotropic distribution of easy axes with similar accuracy. This, of course, does not exclude the possible existence of metastable inhomogeneous states. The fundamental significance of „disorder isotropy“ for occurrence of the Imry–Ma phase has been noted in the paper [57], but a critical value of anisotropy was not calculated in the paper.

For $d = 2$, estimation gives a limitation $\varepsilon < 10^{-1} \div 10^{-2}$ [54] that can be implemented experimentally. An example of a phase diagram of a two-dimensional Heisenberg model with defects of the „random local anisotropy“ type and global anisotropy of the „easy axis“ type [58] is given in Fig. 6. Since the degree of asymmetry of easy axes distribution in the Imry–Ma phase is small, a long-range order cannot exist in it.

In case of anisotropy of the „easy plane“ type, the issue of occurrence of a long-range order or an Imry–Ma state is solved similarly to the case of random fields, only not random fields, but vectors $\mathbf{n}_l$ must be projected onto the „easy plane“.

An example of a non-point defect of the „random anisotropy“ type is the aerogel in ^{3}He–A that affects orientation of the order parameter's orbital portion. The paper [28] describes experimental observation of a transition from the Imry–Ma state into a state with a long-range order

as anisotropy, caused by aerogel deformation, increases. A theoretical estimation of the necessary global anisotropy value for restoration of the long-range order was made in the paper [59].

6. Imry–Ma phase in a nanocrystalline ferromagnet

6.1. Case of nanocrystallites' strong exchange coupling

Another interesting experimentally implementable system with random anisotropy is a nanocrystalline ferromagnet with crystallographic anisotropy of the „easy axis" type. Since crystalline lattice orientation changes from crystallite to crystallite, we have a random anisotropy system.

Anisotropy energy is as follows

$$W_{an} = -\frac{1}{2} K_0 \sum_{\alpha} \sum_{i \in V_\alpha} (\mathbf{s}_{i,\alpha} \mathbf{e}_\alpha)^2, \tag{65}$$

where K_0 is the constant of crystallographic anisotropy in the crystallite volume, $\mathbf{s}_{i,\alpha}$ is the i-th spin pertaining to the crystallite under number α, V_α is the volume of this crystallite, $\mathbf{e}_\alpha$ is the unit vector that sets a direction of the easy axis in the given crystallite.

This section is limited to analysis of the results of consideration of ferromagnets having a high quality factor, where crystallographic anisotropy is much greater than anisotropy of the shape which is not taken into account in this respect.

Such systems were described for the first time in the paper [20], which deals with amorphous ferromagnets where anisotropy of the „easy axis" type changed randomly on the structural correlation length D. Then it was showed, based on the arguments of the Imry–Ma type, that the ferromagnetic state for random anisotropy systems is unstable in relation to occurrence of order parameter static fluctuations.

The papers [21,22] applied this theory to nanocrystalline ferromagnets, where the role of structural correlation length is played by typical crystallite size R. Thereat, is was supposed that exchange interaction between spins of the neighboring crystallites is the same as inside the crystallite. In this case, a domain wall with the thickness of $\Delta \sim \sqrt{J/K_0}$ tries to form on the crystallites boundary, where J is the exchange integral in the crystallite volume. In case of $R \gg \Delta$, such walls form freely, and magnetization in the volume of each crystallite is parallel to its easy axis. This is the region of crystallites' individual behavior.

With $R \ll \Delta$, domain walls cannot form on inter-crystallite boundaries, and the Imry–Ma phase arises, where magnetization changes on the scales $L \gg R$, following the fluctuations in the easy axis direction. Quantity of crystallites in volume L^d is about $(L/R)^d$. Average anisotropy across the volume, arising in the given volume due to statistical fluctuations, is equal to $K_0(R/L)^{d/2}$. The corresponding energy gain per cell is equal to

$$w_1 \approx -K_0 \left(\frac{R}{L}\right)^{d/2}. \tag{66}$$

Inhomogeneous exchange energy is given by the formula (4). The optimal scale of inhomogeneity L^* and an addition to the homogeneous state energy per cell are respectively equal to

$$L^* \approx \Delta \left(\frac{\Delta}{R}\right)^{\frac{d}{4-d}}, \tag{67}$$

$$w_{I-M} \approx -K_0 \left(\frac{R}{\Delta}\right)^{\frac{2d}{4-d}}. \tag{68}$$

The estimate w_{I-M} in case of a random dimension of space was obtained in the paper [60]. This value can be used to estimate the magnitude of coercive field according to the formula [20–22]:

$$H_c \approx \frac{|w_{I-M}|}{M_s}, \tag{69}$$

where M_s is the value of saturation magnetization. This estimate was used in [21] to obtain a power dependence of the three-dimensional sample's coercive field on crystallite size $H_c \propto R^6$.

However, a whole range of alloys have a power dependence with a smaller index: $H_c \propto R^3$ (see, for instance, the monograph [61]). It was explained in the papers [62,63] by means of a model with assumed presence both of random anisotropy and uniaxial anisotropy, homogeneous across the sample volume, which was much greater than the random one. The coercive field magnitude is determined by the total anisotropy energy, consequently, in this case the summand in H_c, proportional R^3, is a small addition to the constant value and cannot cause a considerable change in the coercive field. Therefore, the suggested model contradicts the experimental data which gives evidence of a change in the coercive field by several orders upon a change in crystallite size [64,65].

6.2. Case of nanocrystallites' weak exchange coupling

In order to eliminate the said contradiction, the paper [66] suggested a model of nanocrystallites' weak exchange coupling. This can be related both to the structure of boundaries (large distances between neighboring spins that pertain to different crystallites, presence of amorphous phase) and to a different chemical composition in the crystallites' volume and in the boundary regions.

Let exchange interaction of the nearest (to each other) spins, pertaining to neighboring crystallites, is described by an exchange integral $\tilde{J} \ll J$. If $\tilde{J}$ exceeds the domain wall energy per cell in the wall plane $\varepsilon \sim \sqrt{JK_0}$, then the

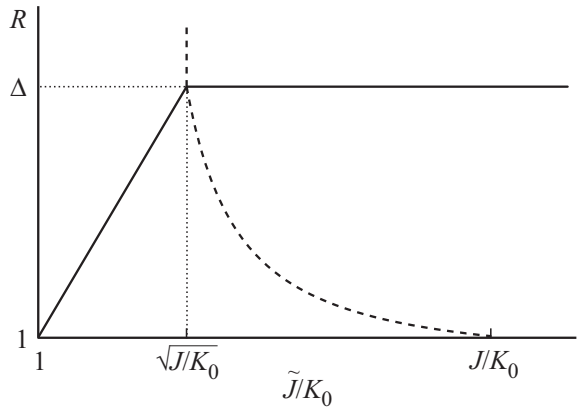

Figure 7. Phase diagram of a nanocrystalline system [66] in the variables „typical crystallite size — exchange integral of intercrystalline interaction“. The solid line is the boundary between regions of individual (above the line) and collective behavior of crystallites. The dashed line separates the regions of the first (to the right of the line) and second mechanisms of magnetization turn.

boundary between crystallites' individual and collective behavior remains unchanged ($R \sim \Delta$). In case of $\tilde{J} \ll \sqrt{JK_0}$, a magnetization turn on the atomic scale in the inter-crystallite gap is energetically more favorable. The critical size $R_{\rm cr}$ is found from the condition of equality of anisotropy energy in crystallite volume K_0R^3 and inhomogeneity energy at the crystallite boundary $\tilde{J}R^2$:

$$R_{\rm cr} \approx \frac{\tilde{J}}{K_0} \approx \frac{\tilde{J}}{\sqrt{JK_0}}\Delta \ll \Delta. \qquad (70)$$

It is to be recalled that all distances are given in units of the spin lattice constant. With $R \gg R_{\rm cr}$ there is individual, and in the opposite case — collective behavior of crystallite magnetizations.

The phase diagram of a system [66] in the variables „typical crystallite size R — exchange integral of intercrystalline interaction“ is given in Fig. 7. The boundary between the individual and collective behavior of crystallite magnetizations is showed by a solid line. Above it there is individual behavior, and below it is collective behavior. A transition from one behavior type to another occurs continuously as the system parameters change.

Type of inhomogeneous exchange energy depends on the mechanism of magnetization turn from one region with linear size L to another. If magnetization turns continuously on scale L (the first turn mechanism), the expression (4) is true.

Magnetization can turn „stepwise“ by successive jumps on crystallite boundaries (the second turn mechanism). The quantity of such steps on length L is equal to L/R, while the turn angle at each crystallite boundary is about R/L. Since inhomogeneity energy is proportional to the turn angle square, inhomogeneity energy in the contact region per cell in the crystallite boundary plane is equal to

$$w_2^{(1)} \approx \frac{\tilde{J}R^2}{L^2}. \qquad (71)$$

The relative fraction of contact regions is about R^{-1}. As a result, the average volumetric density of inhomogeneity energy is

$$w_2 \approx \frac{\tilde{J}R}{L^2}. \qquad (72)$$

By comparing the expressions (4) and (72), we find that the first turn mechanism predominates at $\tilde{J}R \gg J$, and the second one — in case of the inverse ratio.

The boundary between the regions where one of the turn mechanisms prevails is showed in Fig. 7 by a dashed hyperbola $R = \left(\frac{J}{K_0}\right)\left(\frac{\tilde{J}}{K_0}\right)^{-1}$. It is a continuation of the straight line $\frac{\tilde{J}}{K_0} = \sqrt{\frac{J}{K_0}}$ that separates two mechanisms of magnetization turn at the boundaries of large crystallites. A transition from one mechanism to another occurs continuously as the parameters change.

In case of the first turn mechanism, the estimates (67), (68) hold good for the parameters L^* and w_{I-M}, obtained for the case of strong crystallite interaction. For the second turn mechanism, by minimizing the sum of energies (66) and (72) in terms of L, we find [66]:

$$\tilde{L}^* \approx \left(\frac{\tilde{J}}{K_0}\right)^{\frac{2}{4-d}} R^{\frac{2-d}{4-d}}, \qquad (73)$$

$$w_{I-M} \approx -K_0\left(\frac{RK_0}{\tilde{J}}\right)^{\frac{d}{4-d}}. \qquad (74)$$

Dependence $H_c \propto w_{I-M} \propto \tilde{J}^{-\frac{d}{4-d}}$ explains the coercive field growth with temperature increase, found in the paper [67]. Indeed, the value of $\tilde{J}$ drops as the amorphous phase, located in the inter-crystallite gap, approaches the Curie temperature. H_c continues growing up to violation of the condition $K_0R \ll \tilde{J}$ and transition to the phase of individual behavior of crystallite magnetizations.

It is easily seen that dependence of coercive field (in the region of prevalence of the second turn mechanism) on crystallite size ($H_c \propto R^{1/3}$, $H_c \propto R$, $H_c \propto R^3$ for one-dimensional, two-dimensional and three-dimensional structures, respectively) differs significantly from the one in the region of prevalence of the first mechanism ($H_c \propto R^{2/3}$, $H_c \propto R^2$, $H_c \propto R^6$ for one-dimensional, two-dimensional and three-dimensional structures, respectively).

Thus, the dependence $H_c \propto R^3$ observed in the experiment [61,64,65] is explained by the weak inter-crystallite bond and by the stepped mechanism of magnetization turn.

6.3. Texturing degree

The phase diagram [66] given in Fig. 7 is true in the absence of sample texturing, i.e. in the absence of anisotropy in the distribution of easy axes' directions in the order parameter space.

Let us estimate the critical degree ε of asymmetry of distribution $\rho(\mathbf{e})$ of easy axes' directions in the order parameter space, upon exceeding of which the Imry–Ma state becomes energetically unfavorable and a homogeneous ferromagnetic state arises. Let $\rho(\mathbf{e})$ be given by the expressions (36) or (38), in which the components of the random field vector have been replaced by the components of the vector $\mathbf{e}$.

The critical value of ε in case of global anisotropy of the „easy axis" type is equal to

$$\varepsilon_{\text{cr}} \approx \left(\frac{R}{\Delta}\right)^{\frac{2d}{4-d}}, \quad \tilde{J}R \gg J, \tag{75}$$

$$\varepsilon_{\text{cr}} \approx \left(\frac{RK_0}{\tilde{J}}\right)^{\frac{d}{4-d}}, \quad \tilde{J}R \ll J. \tag{76}$$

For $R = 0.1\Delta$ from (75) we obtain $\varepsilon_{\text{cr}} = 10^{-6}$ for $d = 3$ and $\varepsilon_{\text{cr}} = 10^{-2}$ for $d = 2$. For $RK_0 = 0.1\tilde{J}$ we find from (76): $\varepsilon_{\text{cr}} = 10^{-3}$ for $d = 3$ and $\varepsilon_{\text{cr}} = 10^{-1}$ for $d = 2$ [66]. It is easily seen that the least severe requirements to texturing degree refer to two-dimensional samples with crystallites' weak interaction and the second mechanism of magnetization turn. Therefore, the Imry–Ma state is more conveniently studied on thin (with the thickness of one crystallite) nanocrystalline ferromagnetic films, texturized so that the crystallites' crystallographic easy axes are isotropically distributed in the film plane. The substrate should be amorphous, so that it does not create texturing in the film plane.

7. Conclusion

The article by Y. Imry and S.-K. Ma [1] was published 45 years ago. Within this time, the Imry–Ma phase theory, from our point of view, has underwent three stages.

At the first one everything was rather simple. Knowing the space dimension and having made sure that the system has a random field or random anisotropy, the correlation function of which droops at short distances, we were able to conclude that the system ground state corresponds to an unordered Imry–Ma phase, where the order parameter direction follows large-scale fluctuations of a random field or random anisotropy. According to the prevailing viewpoint in science, a random field and random anisotropy destroyed the long-range order.

The beginning of the second research stage has heralded the prediction of the occurrence of a random field induced order ($RFIO$) in a two-dimensional system [10]. All things got mixed up. A long-range order occurred at the end temperature, where it was not present in a pure system. It was realized that random fields may not only destroy but also create a long-range order.

The beginning of the third stage has set the understanding of the role of weak homogeneous anisotropy of the „easy axis" type in suppression of the inhomogeneous Imry–Ma state [57,28,29]. Establishing of the fact of occurrence and finding of a method to calculate the value of global anisotropy, induced by anisotropic distribution of random fields in the order parameter space [17,18,41], made it possible to mark ways for determination of the system state. Only now much more needs to be known than at the first stage, in order to predict a region of Imry–Ma phase existence. Information about the distribution of random fields or easy axes in the order parameter space is necessary. In most cases, the Imry–Ma phase arises in case of relatively strong proximity of this distribution to the isotropic one. While in a theoretical consideration the degree of such proximity can be set by the author, in an experimental study the obtaining of such information, from our point of view, presents severe difficulties. Therefore, we end the overview with a wish of success for enthusiasts who plan to study the very interesting Imry–Ma phases. Their theory, and the more so experimental studies are nowhere near completion.

The two authors (A.A.B. and A.S.S.) deeply lament the untimely death of a talented physicist, friend and co-author — Alexander Igorevich Morozov — and dedicate this article to his cherished memory.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] Y. Imry, S.-k. Ma. Phys. Rev. Lett. **35**, 1399 (1975).
[2] A.I. Larkin. JETP **58**, 1466 (1970).
[3] V.S. Dotsenko. Phys.-Usp. **38**, 457 (1995).
[4] A.I. Morosov, A.S. Sigov. Comments Condens. Matter Phys. **18**, 279 (1998).
[5] L.D. Landau, E.M. Lifshitz. Statisticheskaya fizika. Fizmatlit, M. (2002). 616 p. (in Russian).
[6] N.N. Lebedev, A.P. Levanyuk, A.I. Morozov, A.S. Sigov. Physics of the Solid State **25**, 2975 (1983).
[7] A.P. Levanyuk, A.S. Sigov. Defects and Structural Phase Transitions. Gordon and Breach, N.Y. (1987). 258 c.
[8] T.C. Proctor, D.A. Garanin, E.M. Chudnovsky. Phys. Rev. Lett. **112**, 097201 (2014).
[9] A.A. Berzin, A.I. Morozov, A.S. Sigov. Physics of the Solid State **62**, 281 (2020).
[10] B.J. Minchau, R.A. Pelcovits. Phys. Rev. B **32**, 3081 (1985).
[11] J. Wehr, A. Niederberger, L. Sanchez-Palencia, M. Lewenstein. Phys. Rev. B **74**, 224448 (2006).
[12] N.D. Mermin, H. Wagner. Phys. Rev. Lett. **17**, 1133 (1966).
[13] P.C. Hohenberg. Phys. Rev. **158**, 383 (1967).
[14] D.E. Feldmann. JETP **115**, 2143 (1999).
[15] N. Crawford. J. Stat. Phys. **142**, 11 (2011).
[16] N. Crawford. EPL **102**, 36003 (2013).

[17] A.A. Berzin, A.I. Morozov, A.S. Sigov. Physics of the Solid State **58**, 1614 (2016).
[18] A.A. Berzin, A.I. Morozov, A.S. Sigov. Physics of the Solid State **58**, 1758 (2016).
[19] R.A. Pelcovits, E. Pytte, J. Rudnick. Phys. Rev. Lett. **40**, 476 (1978).
[20] R. Alben, J.J. Becker, M.C. Chi. J. Appl. Phys. **49**, 1853 (1978).
[21] G. Herzer. IEEE Transact. Magn. **25**, 3327 (1989).
[22] G. Herzer. IEEE Transact. Magn. **26**, 1397 (1990).
[23] D.E. Feldmann, R.A. Pelcovits. Phys. Rev. E **70**, 040702(R) (2004).
[24] V.M. Khasanov. JETP Letters **81**, 27 (2005).
[25] L. Petridis, E.M. Terentjev. Phys. Rev. E **74**, 051707 (2006).
[26] A.A. Fedorenko, F. Kuhnel. Phys. Rev. B **75**, 174206 (2007).
[27] J. Elbs, Yu.M. Bunkov, E. Collin, H. Godfrin, G.E. Volovik. Phys. Rev. Lett. **100**, 215304 (2008).
[28] G.E. Volovik. J. Low Temp. Phys. **150**, 453 (2008).
[29] A.I. Morozov, A.S. Sigov. JETP Letters **90**, 818 (2009).
[30] A.A. Berzin, A.I. Morosov, A.S. Sigov. arXiv:2005.08091.
[31] J.F. Fernandez, G. Grinstein, Y. Imry, S. Kirkpatrik. Phys. Rev. Lett. **51**, 203 (1983).
[32] G. Grinstein, S.-k. Ma. Phys. Rev. B **28**, 2588 (1983).
[33] J.Z. Imbrie. Phys. Rev. Lett. **53**, 1747 (1984).
[34] M. Aizerman, J. Wehr. Phys. Rev. Lett. **62**, 2503 (1989).
[35] L. Leuzzi, G. Parizi. Phys. Rev. B **88**, 224204 (2013).
[36] C. Frontera, E. Vives. Phys. Rev. E **59**, R1295 (1999).
[37] S. Sinha. Phys. Rev. E **87**, 022121 (2013).
[38] A.A. Berzin, A.I. Morosov, A.S. Sigov. arXiv:2005.08082.
[39] D.J. Scalapino, Y. Ymry, P. Pincus. Phys. Rev. B **11**, 2042 (1978).
[40] L.D. Landau, E.M. Lifshitz. Elektrodinamika sploshnykh sred. Fizmatlit, M. (2004). 656 p. (in Russian).
[41] A.A. Berzin, A.I. Morozov, A.S. Sigov. Physics of the Solid State **59**, 1992 (2017).
[42] A.I. Morozov. Physics of the Solid State **53**, 705 (2011).
[43] A. Aharony. Phys. Rev. B **18**, 3328 (1978).
[44] V.L. Berezinskii. JETP **59**, 907 (1971).
[45] V.L. Berezinskii. JETP **61**, 1144 (1972).
[46] J.M. Kosterlitz, D.G. Thouless. J. Phys. C **6**, 1181 (1973).
[47] Yu.A. Izyumov, Yu.N. Skryabin. Statisticheskaya mekhanika magnitouporyadochennykh sistem. Nauka, M. (1987). 264 p. (in Russian).
[48] A.Z. Patashinskiy, V.L. Pokrovskiy. Fluktuatsionnaya teoriya fazovykh perekhodov. Nauka, M. (1982). 382 p. (in Russian).
[49] A.A. Berzin, A.I. Morozov. Physics of the Solid State **57**, 2155 (2015).
[50] F. Wegner. Zeitsch. Phys. **206**, 465 (1967).
[51] R.W. Gerling, D.P. Landau. Phys. Rev. B **27**, 1719 (1983).
[52] M.E. Fisher. Am. J. Phys. **32**, 343 (1964).
[53] S.B. Khokhlachev. JETP Letters **70**, 265 (1976).
[54] A.A. Berzin, A.I. Morozov, A.S. Sigov. Physics of the Solid State **58**, 1947 (2016).
[55] A.A. Berzin, A.I. Morozov, A.S. Sigov. Physics of the Solid State **59**, 2420 (2017).
[56] A.I. Morosov, A.S. Sigov. J. Magn. Magn. Mater. **459**, 256 (2018).
[57] I.A. Fomin. JETP Letters **85**, 533 (2007).
[58] A.A. Berzin, A.I. Morozov, A.S. Sigov. Physics of the Solid State **62**, 610 (2020).
[59] I.A. Fomin. JETP Letters **104**, 18 (2016).
[60] G. Herzer. Mater. Sci. Eng. A **133**, 1 (1991).
[61] A.M. Glezer, N.A. Shurygina. Amorfno-nanokristallicheskiye splavy. Fizmatlit, M. (2013). 452 p. (in Russian).
[62] K. Suzuki, J.M. Cadogan. Phys. Rev. B **58**, 2730 (1998).
[63] K. Suzuki, N. Ito, J.S. Garitaonandia, J.D. Cashion, G. Herzer. J. Non-Cryst. Solids **354**, 5089 (2008).
[64] Y. Fujii, H. Fujita, A. Seki, T. Tomida. J. Appl. Phys. **70**, 6241 (1991).
[65] D. Yao, S. Ge, X. Zhou, H. Zuo. J. Appl. Phys. **107**, 073902 (2010).
[66] A.A. Berzin, A.I. Morozov, A.S. Sigov. Physics of the Solid State **60**, 1689 (2018).
[67] A. Hernando, T. Kulik. Phys. Rev. B **49**, 7064 (1994).